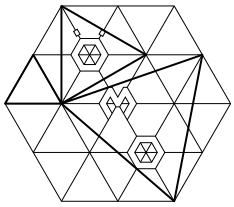

42st Austrian Mathematical Olympiad

Beginner's Competition

June 16th, 2011

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1. Let x be the smallest positive integer such that $2x$ is the square of an integer, $3x$ is the cube of an integer and $5x$ is the fifth power of an integer.

   Find the prime factorization of x.

   S. Wagner, Stellenbosch

2. Let p and q be real numbers such that the quadratic equation

   $$x^2 + px + q = 0$$

   has two real solutions x_1 and x_2.

   The following two conditions hold:

   (i) The numbers x_1 and x_2 differ by 1.

   (ii) The numbers p and q differ by 1.

   Show that p, q, x_1 and x_2 are integers.

   G. Kirchner, Innsbruck

3. Let x, y be positive real numbers such that

   $$x + y + xy = 3.$$

   Prove that

   $$x + y \geq 2.$$

   When does equality hold?

   K. Czakler, Vienna

4. Let ABC be an isosceles triangle with $\overline{AC} = \overline{BC}$ and P be a point of the circumcircle lying on the arc CA not containing B.

   Let E and F be the orthogonal projections of the point C onto the lines AP and BP, respectively.

   Prove that AE and BF have the same length.

   W. Janous, Innsbruck