



37th Austrian Mathematical Olympiad
Beginner's Competition
June 13, 2006

1. Do integers a, b exist such that $a^{2006} + b^{2006} + 1$ is divisible by 2006^2 ?
2. For which real numbers a is the set of all solutions of the inequality

$$(x^2 + ax + 4)(x^2 - 5x + 6) \leq 0$$

an interval?

3. Let n be an even positive integer.

We consider rectangles with integer side lengths k and $k + 1$, where k is greater than $\frac{n}{2}$ and at most equal to n .

Show that for all even positive integers n the sum of the areas of these rectangles equals

$$\frac{n(n+2)(7n+4)}{24}.$$

4. Show that if a triangle has two excircles of the same size, then the triangle is isosceles.

(Note: The excircle ABC to the side a touches the extensions of the sides AB and AC and the side BC .)