



34th Austrian Mathematical Olympiad  
Beginner's Competition  
June 12, 2003

1. Let  $x$  and  $y$  be real numbers such that  $\lfloor \sqrt{x} \rfloor = 10$  and  $\lfloor \sqrt{y} \rfloor = 14$ . What is the value of

$$\left\lfloor \sqrt{\left\lfloor \sqrt{\left\lfloor x+y \right\rfloor} \right\rfloor} \right\rfloor?$$

(Remark: The square roots are all positive.)

2. Determine all real solutions of the equation  $(x - 4)(x^2 - 8 + 14)^2 = (x - 4)^3$ .
3. (a) Show that the product of 5 consecutive even natural numbers is divisible by 15.  
(b) Determine the largest integer  $D$  such that the product of 5 consecutive even natural numbers is always divisible by  $D$ .
4. Show that each rectangle circumscribing a square is itself a square.  
(A rectangle is circumscribing a square if there is exactly one vertex of the square on each side of the rectangle.)