



32nd Austrian Mathematical Olympiad  
Beginner's Competition  
June 7, 2001

1. Prove that for all odd positive integers  $n$  the number  $n^n - n$  is divisible by 24.
2. We consider the quadratic equation  $x^2 - 2mx - 1 = 0$ , where  $m$  is an arbitrary real number.  
For which values of  $m$  does the equation have two real solutions, such that the sum of their cubes equals eight times their sum.
3. Determine all real numbers  $x$  such that the inequality

$$(x - 1)^2(x - 4)^2 < (x - 2)^2$$

holds.

4. Let  $ABC$  be a triangle whose angles  $\alpha = \angle CAB$  and  $\beta = \angle CBA$  are greater than  $45^\circ$ .  
Above the side  $AB$  we construct a right-angled isosceles triangle  $ABR$  with  $AB$  as hypotenuse, such that  $R$  lies *inside* the triangle  $ABC$ .  
Analogously we erect above the sides  $BC$  and  $AC$  right-angled isosceles triangles  $CBP$  and  $ACQ$ , but with their (right-angled) vertices  $P$  and  $Q$  *outside* of the triangle  $ABC$ .  
Show that  $CQRP$  is a parallelogram.