31st Austrian Mathematical Olympiad

Beginner's Competition

June 15, 2000

1. Let a be a real number. Determine for all a all pairs (x, y) of real numbers such that $(x - y^2)(y - x^2) + x^3 + y^3 = a$ holds.

2. Let a and b be positive real numbers. Prove that the inequality

$$\frac{(a+b)^3}{a^2 b} \geq \frac{27}{4}$$

   holds.

   When does equality hold?

3. A "nice" two-digit number is at the same time a multiple of the product of its digits and a multiple of the sum of its digits.

   How many duch two-digit numbers exist?

   What is the quotient of number and sum of digits for each of these numbers?

4. Let $ABCDEFG$ be one half of a regular dodecahedron.

   Let P be the intersection of the lines AB and GF and let Q be the intersection of the lines AC and GE.

   Show that Q is the circumcenter of the triangle AGP.