



# 35th Austrian Mathematical Olympiad

## Regional Competition for Advanced Students

April 20, 2004

1. Determine all integers  $a$  and  $b$  such that  $(a^3 + b)(a + b^3) = (a + b)^4$ .
2. Solve the equation

$$\sqrt{4 - x\sqrt{4 - (x - 2)\sqrt{1 + (x - 5)(x - 7)}}} = \frac{5x - 6 - x^2}{2}$$

over the real numbers.

(All square roots are non-negative.)

3. We are given a convex quadrilateral  $ABCD$  with  $\angle ADC = \angle BCD > 90^\circ$ .  
Let  $E$  be the intersection of the line  $AC$  with the line parallel to  $AD$  through  $B$  and  $F$  be the intersection of the line  $BD$  with the line parallel to  $BC$  through  $A$ .  
Show that  $EF$  is parallel to  $CD$ .
4. The sequence  $\langle x_n \rangle$  is defined by

$$x_{n+1} = \left( \frac{n}{2004} + \frac{1}{n} \right) x_n^2 - \frac{n^3}{2004} + 1 \text{ for } n > 0.$$

Let  $x_1$  be a positive integer smaller than 204 such that all elements of the sequence are positive integers.

Show that the sequence then contains infinitely many primes.