34th Austrian Mathematical Olympiad

Regional Competition for Advanced Students

April 29, 2003

1. Determine the smallest possible value of $\dfrac{a+1}{a(a+2)}+\dfrac{b+1}{b(b+2)}+\dfrac{c+1}{c(c+2)}$ for positive real numbers a, b, c with $a+b+c\leq 3$.

2. Determine all primes p such that 5^p+4p^4 is a square number.

3. We are given two parallel lines g and h and a point P lying outside the stripe between g and h. Three mutually distinct lines g_1, g_2 and g_3 are drawn through P, intersecting g in the points A_1, A_2, A_3 and h in the points B_1, B_2, B_3.
   The points $C_{12}=(A_1B_2)\cap(A_2B_1)$, $C_{13}=(A_1B_3)\cap(A_3B_1)$, $C_{23}=(A_2B_3)\cap(A_3B_2)$ are the intersection points of the corresponding connections.
   Show that

   (a) there exists exactly one line n containing the points C_{12}, C_{13}, and C_{23} and

   (b) n is parallel to g and h.

4. For each real number b determine all real numbers x such that $x-b=\sum_{k=0}^{\infty}x^k$ holds.