



34th Austrian Mathematical Olympiad  
Federal Competition for Advanced Students  
Part 1, May 28, 2003

1. Determine all triples  $(p, q, r)$  of primes such that  $p^q + p^r$  is a square number.
2. Determine the smallest and largest possible value of  $f(x, y) = y - 2x$  for all non-negative real numbers  $x, y$  with  $x \neq y$  and  $\frac{x^2+y^2}{x+y} \leq 4$ .
3. Let  $t$  be a positive real number. Determine the number of positive real solutions  $(a, b, c, d)$  of the following system of equations.

$$a(1 - b^2) = t$$

$$b(1 - c^2) = t$$

$$c(1 - d^2) = t$$

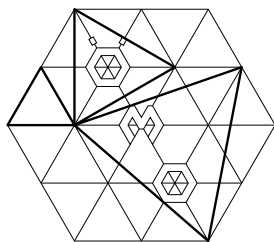
$$d(1 - a^2) = t$$

4. In a parallelogram  $ABCD$ , let  $E$  be the midpoint of the side  $AB$  and  $F$  the midpoint of  $BC$ . Let  $P$  be the intersection point of the lines  $EC$  and  $FD$ .  
Show that the segments  $AP$ ,  $BP$ ,  $CP$  and  $DP$  divide the parallelogram into four triangles with areas in  $1 : 2 : 3 : 4$  ratio.



34th Austrian Mathematical Olympiad  
 Federal Competition for Advanced Students  
 Part 2, Day 1, June 28, 2003

1. Let the polynomial  $P(n) = n^3 - n^2 - 5n + 2$  be given.  
 Determine all integers  $n$  such that  $P(n)^2$  is the square of a prime.
  
2. Let  $a, b, c$  be real numbers different from zero such that there exist  $\alpha, \beta, \gamma \in \{-1, 1\}$  with  $\alpha a + \beta b + \gamma c = 0$   
 What is the smallest possible value of  $\left(\frac{a^3+b^3+c^3}{abc}\right)^2$ .
  
3. Around each grid point  $(x, y)$  with non-negative integers coordinates a square with the grid point as its center and length of its sides equal to  $\frac{0,9}{2^{x5^y}}$  is drawn in arbitrary orientation.  
 Determine the area of this figure consisting of an infinit number of squares.



34th Austrian Mathematical Olympiad  
Federal Competition for Advanced Students  
Part 2, Day 2, June 29, 2003

4. Show that for each base  $g > 2$  there exists exactly one three-digit number  $(abc)_g$ , that is represented as  $(cba)_h$  with the digits in reverse order in a base  $h$  that differs from  $g$  by 1.
5. We are given a sufficient amount of bricks: Rectangles of the size  $2 \times 1$  and squares of the size  $1 \times 1$ .

Let  $n > 3$  be a natural number.

How many possibilities exist to fill a  $3 \times n$  rectangle with these bricks in such a way that all  $2 \times 1$  rectangles have their longer sides parallel to the side of length 3 and do not touch each other?

6. Let  $ABC$  be an acute-angled triangle. The circle  $k$  with diameter  $AB$  intersects the lines  $AC$  and  $BC$  in the points  $P$  and  $Q$ . Let  $R$  be the intersection point of the tangents in  $A$  and  $Q$  and let  $S$  be the intersection point of the tangents in  $B$  and  $P$ .

Show that  $C$  lies on the line  $RS$ .